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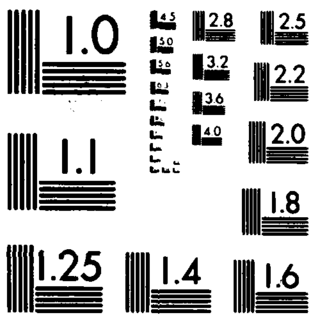
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# The Effect of Surface Energy on Temperature Rise Around a Fast Running Crack

by

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Using a local balance law developed by Gurtin [5] for the cohesive zone, we give a system of equations governing the temperature rise around a fast running crack. We note that the effect of surface temperature cannot be neglected for a fast running crack, and we discuss the equations under simplifying assumptions. In particular, assuming that the surface temperature is much higher than the surrounding temperature, we arrive at a simple solution in closed form. This solution agrees with experimental results of Fuller, Fox,

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and Field [2] for polymethyl methacrylate showing that the temperature rise at the crack tip is independent of crack speed.

## Abstract

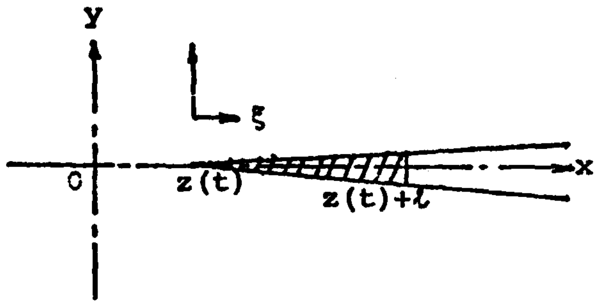
Using a local balance law developed by Gurtin [5] for the cohesive zone, we give a system of equations governing the temperature rise around a fast running crack and discuss the equations under simplifying assumptions.

## 1. Introduction

Using a local balance law developed by Gurtin [5] for the cohesive zone, we give a system of equations governing the temperature rise around a fast running crack. We note that the effect of surface temperature cannot be neglected for a fast running crack, and we discuss the equations under simplifying assumptions. In particular, assuming that the surface temperature is much higher than the surrounding temperature, we arrive at a simple solution in closed form. This solution agrees with experimental results of Fuller, Fox, and Field [2] for polymethyl methacrylate showing that the temperature rise at the crack tip is independent of crack speed.

## 2. Theory

Consideration will be restricted to a semi-infinite crack in an infinite plate stressed symmetrically with respect to the plane of the crack.




We assume that the crack is moving with constant velocity $v$ in the direction of negative $x$. (See the Figure.)

Neglecting thermo-mechanical coupling we have the following energy balance law governing the temperature rise $\theta = \theta(x,y,t)$ away from the crack:

$$\rho c \frac{\partial \theta}{\partial t} = k\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{2.1}$$

where $\rho$ is the density, $k$ conductivity and $c$ specific heat. Let $z(t)$ label the end of cohesive zone and $\ell$ its length, so that $z(t) + \ell$ gives the position of the crack tip.

Since the work rate $f(x,t)$ due to plastic work in the cohesive zone will be symmetric about the plane of the crack, there will be no heat conduction across the line $\{y = 0,\ x < z(t)\}$; hence

$$k \frac{\partial \theta}{\partial y} = 0, \qquad y = 0,\ x < z(t). \tag{2.2}$$

We allow the crack surface to have (surface) temperature rise $\phi(x,t)\,(\geq 0)$, continuous in $x(> z(t))$.

According to a recent theory of Gurtin [5], the local balance law in the cohesive region is given in the form

$$\dot{\epsilon}_f = h + \underset{\sim}{\sigma} \cdot \dot{\underset{\sim}{\delta}}, \tag{2.3}$$

where $h$ is the heat flow per unit surface into the crack surface from the body (cf., e.g., (5.6) in [5]). Using (2.3) we have (by routine assumptions and derivations)

$$\begin{aligned} \frac{\beta}{2}\,\dot{\varphi}(x,t) &= h + \frac{f(x,t)}{2}, \quad y = 0,\ x > z(t), \\ k\,\frac{\partial\theta}{\partial y} &= h, \qquad y = 0^{\pm},\ x > z(t), \end{aligned} \tag{2.4}$$

where $\beta$ is a constant. We assume that $h$ has the form

$$h = \alpha(\theta - \varphi), \tag{2.5}$$

where $\alpha$ (= constant) is the surface conductivity. We note that, in $(2.4)_1$ $f(x,t) = 0$ at $x > z(t) + \ell$, and we neglect heat loss into the surrounding air.[1] In Gurtin [5], the energy of the newly formed free surfaces is assumed to be constant when these surfaces are exposed to a constant environmental temperature. For a fast running crack, however, this assumption is redundant.

Assuming steady-state conditions and introducing the moving coordinate $\xi = x - z(t)$, the equations (2.1)-(2.5) become

[1]See Döll [4].

$$\rho c v \frac{\partial\theta}{\partial\xi} = k\left(\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial y^2}\right), \quad y > 0, \ -\infty < \xi < \infty,$$

$$k\frac{\partial\theta}{\partial y} = 0, \qquad y = 0, \ \xi < 0,$$

$$\frac{\beta}{2} v \frac{\partial\varphi}{\partial\xi} = \alpha(\theta-\varphi) + \frac{g(\xi)}{2}, \quad y = 0, \ \xi > 0, \tag{2.6}$$

$$k\frac{\partial\theta}{\partial y} = \alpha(\theta-\varphi), \qquad y = 0^{\pm}, \ \xi > 0,$$

where $g(\xi) := f(\xi + z(t),t)$ is the work rate in the cohesive region. By $(2.6)_{3,4}$, we have

$$b\frac{\partial\varphi}{\partial\xi} = \frac{\partial\theta}{\partial y} + \frac{g(\xi)}{2k}, \tag{2.7}$$

where $b = \frac{\beta v}{2k}$. We note $\beta$ is the heat required to raise the temperature of a unit surface by one degree (cal/cm$^2$/$^o$C). We expect $\beta$ to be negligible small. The dimensionless constant b, however, is approximately

$6.1\times10^4\times\beta$ - 2024 Aluminium Alloy

$1.7\times10^5\times\beta$ - Mild Steel

$1.4\times10^6\times\beta$ - 6Al-4V Titanium Alloy

$5.0\times10^8\times\beta$ - Polymethyl Methacrylate ($k=5\times10^{-5}$/cm/$^o$C/s)

for $v = 500$ m/sec. Thus, even for $\beta$ as small as $10^{-5}\sim10^{-8}$cal/cm$^2$/$^o$C, we may not neglect the term $b\frac{\partial\varphi}{\partial y}$ compared to $\frac{\partial\theta}{\partial y}$ in (2.7) if the crack velocity is as large as 500 m/s.

We will discuss the temperature rise around a fast running crack using the above equations, but under certain simplifying assumptions.

## 3. Analysis and Discussions

Here we assume that $\theta \ll \varphi$ on the crack surface. (Experiments on polymethyl methacrylate show the temperature rise on the crack faces ($\approx 1\mu m$ in depth) to be about 500 K throughout the velocity range 200~650 m/s [2], in contrast, the maximum temperature rise at a distance of about 0.2~1mm from the crack path, is only 0.1~1 K [3].)

Under this assumption the equations (2.7) become

$$\rho c v \frac{\partial\theta}{\partial\xi} = k\left(\frac{\partial^2\theta}{\partial\xi^2} + \frac{\partial^2\theta}{\partial y^2}\right), \qquad y > 0,\ -\infty < \xi < \infty,$$

$$k\frac{\partial\theta}{\partial y} = 0, \qquad y = 0,\ \xi < 0$$

$$\frac{\beta}{2}\, v\, \frac{\partial\varphi}{\partial\xi} = -\alpha\varphi + \frac{q(\xi)}{2}, \qquad y = 0,\ \xi > 0,$$

$$k\,\frac{\partial\theta}{\partial y} = -\alpha\varphi, \qquad y = 0^{\pm},\ \xi > 0.$$

We assume that $\theta$ and $\varphi$ tend to zero at infinity. We then have solutions in the form

$$\varphi(\xi) = \begin{cases} \displaystyle\int_0^{\ell} \frac{q(t)}{\beta v}\, e^{-\frac{2\alpha}{\beta v}(\xi - t)}\, dt & \ell \le \xi, \\ \displaystyle\int_0^{\xi} \frac{q(t)}{\beta v}\, e^{-\frac{2\alpha}{\beta v}(\xi - t)}\, dt & 0 \le \xi < \ell, \\ 0 & \xi < 0, \end{cases}$$

$$\theta(\xi, y) = \int_0^{\infty} \frac{\alpha\varphi(t)}{k}\, K(\xi - t, y)\, dt,$$

where

$$K(x,y) = \frac{1}{\pi}\, e^{ax} K_0\left(a\sqrt{x^2+y^2}\right), \qquad a = \frac{\rho c v}{2k},$$

with $K_o$ the modified Bessel function of the second kind.

Using the Dugdale model, the work rate due to plastic work is given in the form[1]

$$g(x) = \sigma_o \frac{d\delta(x)}{dx} v,$$

where $\sigma_o$ is the yield stress and $\delta$ the separation distance. Thus the temperature rise $\varphi(\xi)$ for $\xi \geq \ell$ becomes

$$\begin{aligned}\varphi(\xi) &= \left(e^{-\frac{2\alpha\ell}{v\beta}} \int_0^{\ell} \frac{\sigma_o \frac{d\delta(t)}{dt}}{\beta} e^{\frac{2\alpha}{v\beta}t} dt\right) e^{-\frac{2\alpha}{v\beta}(\xi-\ell)} \\ &= \left(e^{-\frac{2\alpha\ell}{v\beta}} \int_0^{1} \frac{\sigma_o \frac{d\delta(\ell x)}{dx}}{\beta} e^{\frac{2\alpha\ell}{v\beta}x} dx\right) e^{-\frac{2\alpha}{v\beta}(\xi-\ell)}\end{aligned} \tag{3.1}$$

and thus, if $2\alpha\ell/v\beta$ is negligibly small, (3.1) is approximately,

$$\varphi(\xi) \approx \frac{\sigma_o \delta(\ell)}{\beta} e^{-\frac{2\alpha\eta}{\beta v}}, \tag{3.2}$$

where $\eta = \xi - \ell$.

This result shows, interestingly, that <u>the temperature rise at the crack tip</u> ($\eta = 0$) <u>is proportional to</u> $\sigma_o\delta(\ell)$ <u>with constant of proportionality is</u> $1/\beta$.[2]

---

[1] The work rate $g(\xi)$ for the Dugdale model in small plane strain is computed by Levy and Rice [1] as

$$g(\xi) = \frac{4(1-\nu^2)\sigma_o^2 v}{E} \ell n\left(\frac{1+\sqrt{\xi/\ell}}{1-\sqrt{\xi/\ell}}\right), \quad 0 < \xi < \ell.$$

[2] Note that we have assumed plastic work in the cohesive zone is completely converted into heat. (This is generally true of the energy expended in plastically deforming a metal.) For partial conversion $g$ should be scaled down appropriately.

Note that $\varphi$ depends on $v$ only through a dependence on $\eta/v$. For the point $x = y = 0$, say, occupied by the tip at $t = 0$, $\eta$ will be the distance from the tip at time $t = \eta/v$. Thus, by (3.2), the temperature rise at this point should depend only on time; it should be independent[1] of $v$. This result is in agreement with experimental result of Fuller, Fox, and Field [2], who found that the temperature rise at a fixed point on the axis of the crack was approximately independent of crack velocity. In fact, they note that "the results combined to give a temperature rise of approximately 500 K throughout the velocity range studied (200~650 m/s)."

For the fixed point $x = y = 0$, (3.2) gives

$$\varphi \approx \frac{\sigma_o \delta(\ell)}{\beta} e^{-\frac{2\alpha}{\beta}t}$$

and hence $\varphi$ has the form

$$\varphi \approx Ae^{-Bt},$$

where $A = \sigma_o \delta(\ell)/\beta$ and $B = -2\alpha/\beta$.

The data of [2], when averaged, give

$$\varphi = 457 \text{ K} \quad \text{at} \quad t = 10\mu s$$

$$\varphi = 361 \text{ K} \quad \text{at} \quad t = 20\mu s$$

$$\varphi = 304 \text{ K} \quad \text{at} \quad t = 35\mu s.$$

Using the values at $t = 10\mu s$ and $t = 35\mu s$, we find that

$$A = 5.38\times10^2 \text{K},$$

$$B = 1.63\times10^4 \text{sec}^{-1}.$$

---

[1] Here we follow Levy and Rice [1] and assume that $\sigma_o \delta(\ell)$ is independent of $v$. Levy and Rice [1], however, found that this temperature rise is proportional to $\sqrt{v}$.

This gives $\varphi = 388$ K at $t = 20\mu s$ as compared to the value $\varphi = 361$ K of [2]. Further, we find that $\varphi = 538$ K at $t = 0$; that is, the temperature rise at the crack tip is 538 K. For the values $\ell = 1$ mm (which we feel is an upper bound for $\ell$) and $v = 200$ m/s, we find that the dimensionless constant

$$2\alpha\ell/v\beta = \ell B/v,$$

which we neglected in defining (3.2), has the approximate value of 0.08.

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